S. 80 Nr. 8

$$f(t) = \frac{at^2 + bt + c}{t^2 + d} \qquad t \geq 0$$

a) $f(0) = 12 \qquad 12 = \frac{c}{d}$

$f(1) = 4{,}5 \qquad 4{,}5 = \frac{a + b + c}{1 + d}$

$f(2) = 6 \qquad 6 = \frac{4a + 2b + c}{4 + d}$

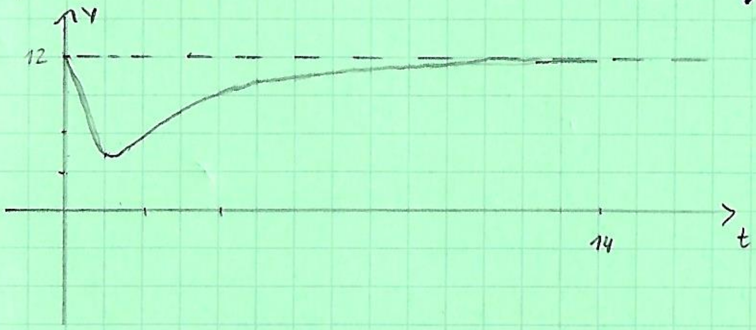
$f(3) = 7{,}5 \qquad 7{,}5 = \frac{9a + 3b + c}{9 + d}$

$$\begin{array}{lrcl} \text{I} & 0 & = & c - 12d \\ \text{II} & 4{,}5 & = & a + b + c - 4{,}5d \\ \text{III} & 24 & = & 4a + 2b + c - 6d \\ \text{IV} & 67{,}5 & = & 9a + 3b + c - 7{,}5d \end{array}$$

$a = 12 \quad b = -15 \quad c = 12 \quad d = 1$

$$f(t) = \frac{12t^2 - 15t + 12}{t^2 + 1}$$

b) Asymptote: $y = 12 \qquad T(1; 4{,}5)$



c) 90 % von $12 \frac{mg}{L}$: $10{,}8 \frac{mg}{t}$

$f(t) = 10{,}8$ GTR Graph: x-cal $t_1 = 0{,}08$ usw.

$t_2 = 12{,}4$ Tage

S. 86 Nr. 24

$$f(x) = \frac{ax}{x^2 + b} \qquad x \geq 0$$

a) 1. $f(100) = 25 \qquad \frac{100a}{10^4 + b} = 25$

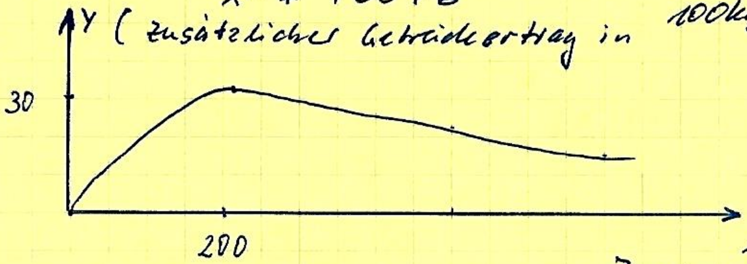
2. $f(600) = 20 \qquad \frac{600a}{36 \cdot 10^4 + b} = 20$

$$\begin{array}{ll} \text{I} & 100a - 25b = 10^4 \cdot 25 \\ \text{II} & 600a - 20b = 20 \cdot 36 \cdot 10^4 \end{array}$$

$a = 13461{,}5 \approx 13462$

$b = 43846{,}2 \approx 43846$

b) $f(x) = \frac{13462x}{x^2 + 43846}$



$P_{MAX}(209{,}4 / 32{,}1)$

~ optimale Düngerzugabe:

$209{,}4 \frac{kg}{ha}$

dann $32{,}1 \cdot 100 \frac{kg}{ha}$ zusätzlicher Ertrag

$32100 \frac{kg}{ha}$

S. 100 Nr. 4

a) symmetrisch zum Ursprung ≙ ungerade Funktion

$f(x) = ax^5 + bx^3 + cx \qquad f'(x) = 5ax^4 + 3bx^2 + c \qquad f''(x) = 20ax^3 + 6bx$

1) $f(-1) = 1$ $\qquad$ I $\; -a - b - c = 1$

2) $f'(-1) = 3$ $\qquad$ II $\; 5a + 3b + c = 3$ $\quad +$

3) $f''(-1) = 0$ $\qquad$ III $\; -20a - 6b = 0$

$$\begin{aligned} 4a + 2b &= 4 \quad | \cdot 3 \quad + \\ -20a - 6b &= 0 \\ \hline -8a &= 12 \\ a &= -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} 4 \cdot \left(-\frac{3}{2}\right) + 2b &= 4 \\ 2b &= 10 \\ b &= 5 \end{aligned}$$

$$\begin{aligned} \frac{3}{2} - 5 - c &= 1 \\ c &= -\frac{9}{2} \end{aligned}$$

$$f(x) = -\frac{3}{2}x^5 + 5x^3 - \frac{9}{2}x$$

b) $f(x) = ax^4 + bx^2 + c \qquad f'(x) = 4ax^3 + 2bx \qquad f''(x) = 12ax^2 + 2b$

1. $f(1) = 0$ $\qquad$ I $\; a + b + c = 0$

2. $f'(1) = 1$ $\qquad$ II $\; 4a + 2b = 1$ $\quad | \quad -$

3. $f''(1) = 0$ $\qquad$ III $\; 12a + 2b = 0$

$$\begin{aligned} -8a &= 1 \\ a &= -\frac{1}{8} \\ -\frac{1}{2} + 2b &= 1 \\ b &= \frac{3}{4} \\ -\frac{1}{8} + \frac{3}{4} + c &= 0 \\ c &= -\frac{5}{8} \end{aligned}$$

$$f(x) = -\frac{1}{8}x^4 + \frac{3}{4}x^2 - \frac{5}{8}$$

lok. Extrempunkte: $-\frac{1}{2}x^3 + \frac{3}{2}x = 0$

$x_1 = 0$

$x_{2|3} = \pm\sqrt{3}$

$f''(0) = \frac{3}{2} > 0 \rightsquigarrow P_{Min}\left(0 \middle| -\frac{5}{8}\right)$

$f''(\pm\sqrt{3}) = -\frac{9}{2} + \frac{3}{2} < 0$

$f(\sqrt{3}) = -\frac{9}{8} + \frac{9}{4} - \frac{5}{8} = \frac{1}{2}$

$\rightsquigarrow P_{Max_1}\left(\sqrt{3} \middle| \frac{1}{2}\right)$

$P_{Max_2}\left(-\sqrt{3} \middle| \frac{1}{2}\right)$

S. 33 **8** g) $f'(x) = 2x^5 + \frac{24}{7}x^3 + \frac{6}{5}x^2$; $f''(x) = 10x^4 + \frac{72}{7}x^2 + \frac{12}{5}x$;
$f'''(x) = 40x^3 + \frac{144}{7}x + \frac{12}{5}$

h) $f'(x) = \frac{15}{2}x^8 - \frac{9}{2}x^5 + \frac{2}{3}x^2$; $f''(x) = 60x^7 - \frac{45}{2}x^4 + \frac{4}{3}x$;
$f'''(x) = 420x^6 - 90x^3 + \frac{4}{3}$

i) $f'(x) = \frac{45}{7}x^8 + 30x^5 - \frac{35}{3}x^4$; $f''(x) = \frac{360}{7}x^7 + 150x^4 - \frac{140}{3}x^3$;
$f'''(x) = 360x^6 + 600x^3 - 140x^2$

9 a) $f'(x) = \frac{2}{3}\left(\frac{1}{3}x + 2\right)$ b) $f'(x) = (3x+2)^5$ c) $f'(x) = -\frac{7}{4}x\left(\frac{1}{2} - x^2\right)^6$
d) $f'(x) = -2(3-x)$ e) $f'(x) = 2(2x+1)(x+x^2)$
f) $h'(x) = 3(2-3x+x^2)^2(2x-3)$ g) $h'(x) = 2(1-x+x^3)(3x^2-1)$
h) $f'(x) = 2(x\sqrt{2} - x^2)(\sqrt{2} - 2x)$ i) $g'(x) = 6x[1 + (x^2-1)^2]$

10 a) $f'(x) = -8(8x-7)^{-2}$ b) $f'(x) = +2(5-x)^{-2}$ c) $f'(x) = -30(15x-3)^{-3}$
d) $f'(x) = -3\left(\frac{1}{2}x - 5x^3\right)^{-4}\left(\frac{1}{2} - 15x^2\right)$ e) $f'(x) = -2(x-2)^{-3}$
f) $g'(t) = +6(5-t)^{-3}$ g) $f'(x) = -\frac{3}{2}(x-7)^{-4} = -\frac{3}{2(x-7)^4}$
h) $g'(t) = -20t(t^2-1)^{-3}$ i) $f'(x) = 4(2x+1) - 4(2x+1)^{-3}$

11 a) $f'(x) = \sqrt{\frac{3}{4x}} = \frac{3}{2\sqrt{3x}}$ b) $f'(x) = \frac{1}{\sqrt{1+2x}}$ c) $g'(x) = \frac{-x}{\sqrt{1-x^2}}$

12 a) $f'(r) = \frac{7-2r}{2\sqrt{7r-r^2}}$ b) $g'(a) = \frac{-15(a^2-2a)}{(a^3-3a^2)^2}$
c) $h'(t) = -5(7-4t^3)(7t-t^4)^{-6}$
d) $f'(x) = -2(x^3 - \sqrt{x})^{-3}\left(3x^2 - \frac{1}{2\sqrt{x}}\right) = (x^3 - \sqrt{x})^{-3}\left(\frac{1}{\sqrt{x}} - 6x^2\right)$
e) $f'(x) = 3x^2 + \frac{2x+1}{2\sqrt{x^2+x}}$ f) $f'(x) = \frac{1}{2}(x^4 - 3x^2 + 1)^3(2x^3 - 3x)$
g) $f'(x) = -\frac{3x^2-2x}{(x^3-x^2+6)^2}$ h) $f'(x) = -\frac{2}{x^3} - \frac{3x^2}{2\sqrt{5-x^3}}$ i) $f'(x) = -\frac{1}{4\sqrt{4x - x\sqrt{x}}}$

13 a) $f'(x) = 12(4x-7)^2$; $f''(x) = 96(4x-7)$
b) $f'(x) = 3(5-x)^{-2}$; $f''(x) = 6(5-x)^{-3}$
c) $f'(x) = -3(x-5)^{-4}$; $f''(x) = 12(x-5)^{-5}$

14 a) $f'(x) = \sqrt{x} + \frac{x}{2\sqrt{x}} = \frac{3}{2}\sqrt{x}$ b) $f'(x) = 2x\sqrt{x} + \frac{x^2}{2\sqrt{x}} = \frac{5}{2}x\sqrt{x}$
c) $f'(t) = 3\sqrt{t} + \frac{3t+2}{2\sqrt{t}} = \frac{9}{2}\sqrt{t} + \frac{\sqrt{t}}{t} = \sqrt{t}\left(\frac{9}{2} + \frac{1}{t}\right)$
d) $g'(t) = 4t\sqrt{t} + \frac{2t^2-3}{2\sqrt{t}} = 5t\sqrt{t} - \frac{3\sqrt{t}}{2t} = \sqrt{t}\left(5t - \frac{3}{2t}\right)$
e) $f'(a) = \frac{1-2a^3}{2\sqrt{a}} - 6a^2\sqrt{a} = \frac{\sqrt{a}}{2a} - 7a^2\sqrt{a} = \sqrt{a}\left(\frac{1}{2a} - 7a^2\right)$
f) $a'(t) = \frac{1+t}{2\sqrt{t}} + \sqrt{t} = \frac{\sqrt{t}}{2t} + \frac{3}{2}\sqrt{t} = \sqrt{t}\left(\frac{1}{2t} + \frac{3}{2}\right)$

S. 33 **15** a) $f'(x) = 1 - 12x$
$f(x) = 1 + x - 6x^2$
b) $f'(x) = 5x^4 - 12x^2 + 2x$
$f(x) = x^5 - 4x^3 + x^2 - 4$
c) $f'(x) = 2x - 4x^3$
$f(x) = x^2 - x^4$

16 a) $f'(x) = \sqrt{x} + \frac{x+k}{2\sqrt{x}} = \frac{3x+k}{2\sqrt{x}}$
b) $f'(x) = 2 - x^n + n x^{n-1} - n x^n$
c) $f'(x) = \frac{3kx+1}{2\sqrt{x}}$
d) $f'(x) = \frac{x-t}{2\sqrt{x}} + \sqrt{x} = \frac{3x-t}{2\sqrt{x}}$
e) $f'(t) = -\sqrt{x}$
f) $f'(k) = 0$

17 a) $f'(x) = 2x\,g(x) + x^2 g'(x)$
$f''(x) = 2g(x) + 4x\,g'(x) + x^2 g''(x)$
b) $f'(x) = g'(x) + x\,g''(x)$
$f''(x) = 2g''(x) + x\,g'''(x)$
c) $f'(x) = (g'(x))^2 + g(x)\cdot g''(x)$
$f''(x) = 3g'(x)g''(x) + g(x)g'''(x)$

18 a) $f'(x) = 2(3x+4)(9x+1)$
b) $f'(x) = (5-4x)^2(16x-17)$
c) $f'(x) = 2(2x+3)^2(2x-1)(10x+3)$
d) $f'(x) = \frac{3-9x}{2\sqrt{3x}}$
e) $f'(x) = \frac{1+2x^2}{\sqrt{1+x^2}}$
f) $f'(x) = \frac{-6x^5 - 4x^3 + 21x^2 + 7}{2\sqrt{(x^2+1)(7x-x^4)}}$

19 a) $f'(x) = -3\cdot a^{2-3x}\cdot \ln(a)$
b) $f'(x) = -2^{3x} + 3(1-x)\cdot 2^{3x}\cdot \ln(2)$
c) $f'(x) = 3a^2(x-1)^2$
d) $f'(x) = ((\ln(3))x + 1)\cdot 3^x$
e) $f'(x) = \frac{3-x}{2\cdot\sqrt{x^5}}$
f) $f'(x) = \frac{\left(\ln(a) - \frac{a}{x}\right)a^x}{x^a}$

S. 34 **20** a) $f'(x) = \frac{1}{(x+1)^2}$ b) $f'(x) = \frac{3}{(2x+3)^2}$ c) $f'(x) = \frac{2}{(1+3x)^2}$ d) $f'(x) = \frac{-6}{(3+x)^2}$

21 a) $f'(x) = \frac{x^2-4}{(x^2+4)^2}$ b) $f'(x) = \frac{-x^2-4x-1}{(x+2)^2}$ c) $f'(x) = \frac{2x}{(1+3x^2)^2}$ d) $f'(x) = \frac{-x^2-4x-1}{(x^2-1)^2}$

22 a) $g'(x) = \frac{26x}{(x^2+4)^2}$ b) $g'(t) = \frac{-12t^2}{(2+t^3)^2}$ c) $f'(t) = \frac{-t^4-3t^2-2t}{(t^2+1)^2}$ d) $h'(r) = \frac{4r^5-8r^3}{(r^2-1)^2}$

23 a) $f'(x) = \frac{90+6x^2}{(15-x^2)^2}$ b) $f'(t) = 2\cdot\frac{4t^2+4t+5}{(2t+1)^2}$ c) $h'(a) = 2\cdot\frac{3a^3-6a^2-4}{(3a-4)^2}$ d) $z'(t) = \frac{0{,}8t^2+2t-1{,}5}{(1+0{,}8t)^2}$

24 a) $f'(x) = \frac{1-x}{2\sqrt{x}(x+1)^2}$ b) $f'(x) = \frac{-1}{(\sqrt{x}-1)^2\cdot\sqrt{x}}$ c) $g'(x) = \frac{-1}{\sqrt{x^2-1}\cdot(x-1)}$
d) $x > 0$: $f'(x) = \frac{x-1}{(x+1)^2\cdot\sqrt{x^2+1}}$; $x < 0$: $f'(x) = \frac{x+1}{(1-x)^2\cdot\sqrt{x^2+1}}$

25 a) $f(x) = (x^2-5)^{-1}$; $f'(x) = -2x(x^2-5)^{-2} = \frac{-2x}{(x^2-5)^2}$
b) $f'(x) = \frac{16x}{(7-2x^2)^2}$ c) $f'(x) = \frac{-1}{2\sqrt{x}(\sqrt{x}-x)^2}$ d) $f'(x) = \frac{-4x+8}{(x^2-4x)^2}$
e) $f(x) = x^2+1$; $f'(x) = 2x$ f) $f(x) = \frac{3}{2} - \frac{1}{2x}$; $f'(x) = \frac{1}{2x^2}$ g) $f'(x) = \frac{3}{4} + \frac{5}{4x^2}$
h) mit Polynomdivision: $f(x) = 2x + 1 + \frac{1}{x+1}$; $f'(x) = 2 - \frac{1}{(x+1)^2}$

26 a) $f'(x) = \frac{16x^3}{(x^4+4)^2}$ b) $f'(x) = \frac{-16}{x^5}$ c) $f'(x) = \frac{x^2+2x}{(x+1)^2}$ d) $f'(x) = \frac{-4(2x+1)}{(x^2+x)^2}$
e) $f'(x) = \frac{-x^2+8x+20}{(4-x)^2}$ f) $f'(x) = \frac{1}{4}$ g) $f'(x) = 2\cdot\frac{x^2+x-1}{(2x+1)^2}$ h) $f'(x) = \frac{1}{2}$